ON THE EXPERIMENTAL DETERMINATION OF THE MINIMUM OIL FILM THICKNESS IN A PLAIN JOURNAL BEARING

W. D. BROTHERTON, JR.

Released by Committee 1/27/69

ON THE EXPERIMENTAL DETERMINATION OF THE MINIMUM OIL FILM THICKNESS IN A PLAIN JOURNAL BEARING

-

William D. Brotherton, Jr.

ON THE EXPERIMENTAL DETERMINATION OF THE MINIMUM OIL FILM THICKNESS IN A PLAIN JOURNAL BEARING

by

William DeRoy Brotherton, Jr.
Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE
in
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California
1952

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE
in
MECHANICAL ENGINEERING

from the

United States Naval Postgraduate School.

Paul J. Kiefer
Chairman
Department of Mechanical Engineering.

Approved:

Academic Dean

ACKNOWLEDGMENTS

The writer wishes to express his appreciation to Professor E. K. Gatcomb for his suggestions and guidance throughout the preparation of the paper, to Dr. M. D. Hersey (of Engineering Experiment Station, Annapolis) for his inspirational assistance during initial formulation, and to Professor P. E. Cooper for guidance on the fringe counting mechanism.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

Symbols	Name	Units
	Revolutions per minute	RPM
P	Load per unit projected area	psi
μ	Viscosity	$\frac{\text{lb-sec}}{\text{IN}^2}$
h_{min}	Minimum film thickness	IN
c	Radial clearance	IN
C	Diametrical clearance	IN
D	Journal diameter	IN
L	Bearing length	IN

INTRODUCTION

The primary object of this paper is to make a survey of some of the experimental methods which have been employed to determine the minimum oil film thickness in an operating journal bearing with the intent of recommending the best method to be used by further investigators. A secondary objective is to include as many as possible of the various methods since their descriptions are widely dispersed throughout the literature.

The modern tendency toward the use of high-speed machines with heavy load concentrations on the bearings makes it essential to know just what this minimum film thickness is in order to properly design compact bearings that will give long and dependable service under adverse as well as desirable operating conditions.

It might be said that the existence of film lubrication was accidentally discovered by Tower (1) in his experimente with a bath lubricated half bearing. This discovery led to the study of lubrication as a particular problem in fluid motion. Reynolds (2) arrived at the differential equation for the lubrication of a bearing. Sommerfeld (3) succeeded in integrating Reynold's equation for all values of shaft eccentricity and in extending the solution to the half and the full bearing, keeping Reynold's assumptions of negligible side leakage (an infinitely long bearing) and regarding the viscosity of the lubricant as constant. From this point, no mathematical solution, for all ranges, which has considered side leakage has been forthcoming although many approximate solutions have been proposed. The solution of Reynold's equation, including side leakage, has been worked out exactly in certain

ranges for full and partial bearings subjected to constant load by Muskat and Morgan (5), by Cameron and Woods (6), and by Waters (7). A solution of the problem considering both side leakage and variable viscosity was achieved by Kingsbury (4) with the aid of an electrical analogy.

DESCRIPTION OF SOME METHODS USED

In 1916 Gümbel (10) made one of the first attempts to determine the shaft eccentricity by means of two levers arranged at right angles. The results were not satisfactory, however, on account of vibration. Stoney, Boswall, and Massey (11) and Boswall and Brierley (8) made some measurements using an apparatus designed by Dr. Gerald Stoney. This apparatus consisted of a journal which worked in conjunction with two diametrically opposed bearings carried by two vertical arms. The arms are coupled together by two independent links each comprising a bolt with knife-edge attachments. The distance between the lower pair of knife-edges was fixed. These points act as centers about which the arms can rotate, but place no restriction upon small parallel displacements of the arms in a vertical direction. The upper pair of knife-edges enables pressure to be applied on the arms at these points by means of a spring which can be compressed by a wing-nut. For the purpose of measuring displacements of the bearings relative to the journal, two sensitive micrometers, one vertical and the other horizontal, are fitted at the upper end of the arms. The accuracy of the measurements is increased by the length of the lever arms to which the micrometers are attached.

Commencing in about 1916, a group of students under the direction of Professor G. H. Marx (12) at Stanford University conducted a series of experiments with lightly loaded bearings using a screw-micrometer arrangement (three micrometers equally spaced around the journal). The stems of these micrometers were passed through the bearing and formed part of a series electrical circuit with the journal, earphones, and a small dry cell. With this setup, the earphones gave a distinct click when the stems

of the micrometers were screwed into contact with the journal. The results of these experiments indicated that the journal tended to ride slightly above the center of the bearing.

In 1929 Goodman (13) published the results of tests using two Geneva Gages spaced at ninety degrees. These gages were mounted in a cage secured to the shaft by two pre-loaded ball bearings, one on each side of the test bearing. The ends of the gages then rested against the outside of the bearing shell, their readings thus gave the horizontal and vertical movement of the center of the journal with respect to the bearing.

Bradford and Davenport (14) give results when using a machine (complete description is given in Bulletin No. 39 of the Engineering Experiment Station of The Pennsylvania State College) which had three equally spaced dial micrometers fitted to the end of the bearing and having their stems bearing against the shaft.

In 1930 Kluge and Linckh (15) made some measurements by use of piezo-electric methods. The principle of this method utilizes the property of a crystal of quartz to charge up electrically when it is subjected to forces which attempt to deform the crystal.

Stone (16) used an electromagnetic gage method which consists of mounting two U-shaped electremagnets diametrically opposite each other, with a ring of laminations shrunk on the shaft forming the armature. The electromagnets carry a primary and a secondary winding — the primaries connected in series, the secondaries in series opposed. For a central position of the shaft, the voltage in the secondary circuit is zero. As the shaft moves, effectively changing the reluctance of the circuit by increasing the air gap on one side and decreasing it on the other, the

secondary voltage rises directly with the motion.

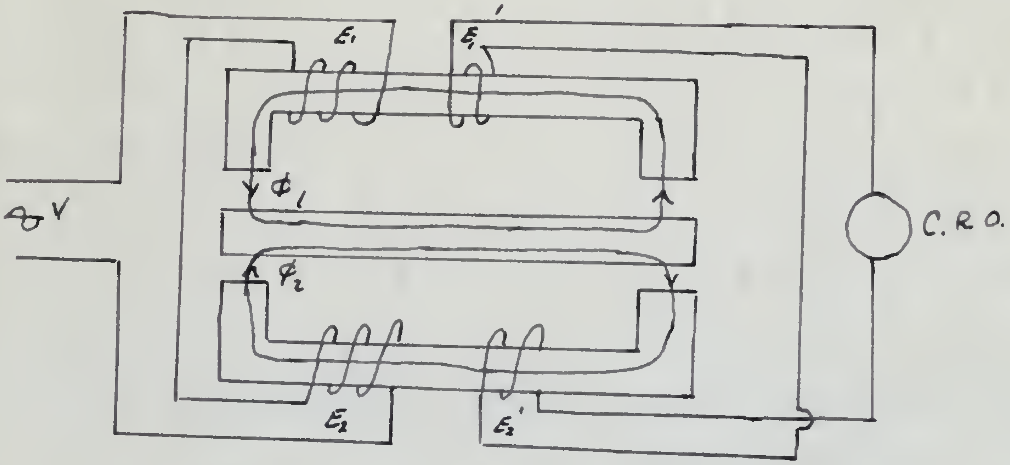


Fig. 1. Schematic of Stone's Electromagnetic Micrometer

The experimental apparatus has a claimed accuracy to less than 1/100,000 inches. For a slight movement of the armature (shaft), an appreciable value of $E_2' - E_1'$ is obtained which is a direct measure of the shaft movement. Calibration is obtained by measurement of the voltage trace for a known displacement. The shaft movement is then obtained by measuring the voltage trace and multiplying by the calibration factor. By using two sets of these measuring coils, the motion of the shaft center can be determined.

Stone and Underwood (17) measured the minimum film thickness for a rotating load by passing a pin through the bearing and holding it in place against the shaft by a leaf spring. This pin in turn was fastened to a movable plate of a capacitor. The change in capacity is thus a measure of the film thickness.

Simons (18) used a capacitive micrometer (details of circuit given in Electronics Vol.19, 1946, pp 106-111) which consists of two capacitor probes mounted at right angles which will show the position of the journal with reference to a fixed point. In principle, minute displacements of the shaft are measured as a function of changes in electrical capacitance between the shaft and the micrometer probes. This capacitance is made part of the resonant circuit of a high-frequency radio oscillator, and variations cause sufficient changes in oscillator frequency to be readily measured by techniques developed for frequency-modulation broadcasting.

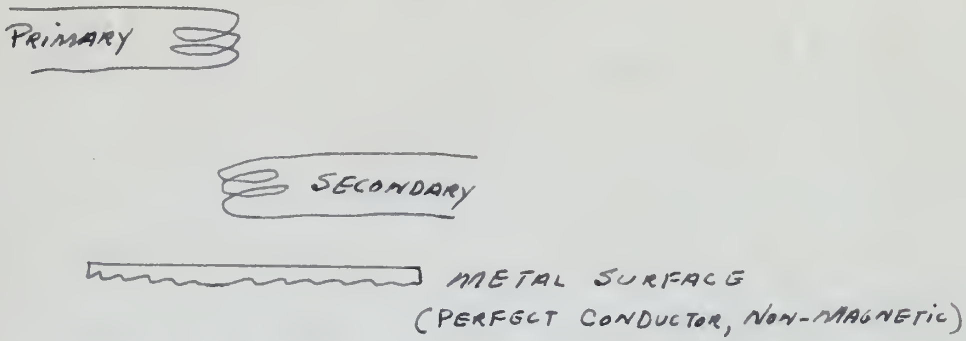
Physically, the apparatus uses two probes lapped to the same radius as a short shaft extension secured outside the bearing. These elements form essentially a split-stator capacitor whose rotor is the shaft extension. Each micrometer channel output is connected normally to one pair of plates of an oscilloscope. The pattern produced on the oscilloscope screen by rotation of the shaft represents the position of the shaft axis.

As used by Simons, the oscilloscope screen is used as the clearance circle, that is, a circle whose radius is the radial clearance between the shaft and bearing. Starting with the spot on the scope at the rest position of the shaft when the shaft is not in motion, the motion of the spot will thus represent the motion of the shaft center as the shaft comes

up to speed and equilibrium is reached.

The instrument is calibrated by measuring the spot deflection on the scope for a known shaft displacement. With this factor, the shaft eccentricity can be determined by making measurements on the scope itself or on a photograph of the scope.

Greengough (19) has experimented with a mutual inductance type of distance measuring element which was developed on the principle of variations of mutual inductance between coupled air-core coils excited at radio frequency.



The primary coil is excited at radio frequency — the plane of the coil is parallel to the plate. Under these conditions the electromagnetic field at the surface of the plate is exactly cancelled by the field of the eddy currents induced in the plate. A secondary or probe coil placed just at the surface would have no voltage induced in it. If the probe coil is moved away from the plate toward the exciting coil, it will be found that an increasing voltage is picked up as the probe coil is moved closer to the primary coil. The voltage output of the probe coil can then be used as an indication of the distance between it and the metal surface.

To eliminate mechanical difficulties, both coils are mounted on one form, and this assembly moved with respect to the metal. The instrument, as used to measure shaft eccentricity, consists of four probes and

associated electrical circuits mounted ninety degrees apart around the shaft. The base plate is a one and one-half inch wide band of copper electroplated on the shaft just outside the bearing area.

By applying the voltages from the probes to a cathode-ray screen and employing the circuits described in the basic paper, the spot on the screen is an accurate reproduction of the shaft eccentricity. The method is said to be substantially independent of the dielectric constant of whatever insulating material is placed between the probes and the metal surface. Calibration is said to be quite simple, although provision must be made in the bearing mounting to move the shaft in the bearing by means of a hoist or jacks. The shaft is held against the bearing wall immediately under each probe in turn. The zero-set control for each probe is then adjusted so that the spot on the cathode-ray tube is at the center of the screen. Since the shaft-bearing clearance is known precisely, this figure will be the spacing between the shaft and the bearing at the location diametrically across from the point of contact of shaft and bearing. The single probe deflection factor is one-half, so that the control knob is manipulated for an indication of one-half this total clearance. When these adjustments have been made for all four probe assemblies, the instrument is completely calibrated. This method also uses the scope as the clearance circle, a given displacement of the shaft center is known to give a known displacement on the scope from which the actual shaft eccentricity can be determined.

Tudor (20) made a study of bearing lubrication utilizing the electrical conductance between the shaft and bearing. Employing a cathode-ray oscillograph as an indicator and a moving film camera to record the conductance variation, he had some success in getting an indication of

variations in film thickness (the conductance measurements were carried out by a potentiometric method). For low values of voltage across the oil film, the current-voltage curve was linear which indicated constant film conductance. As the potential across the film was increased, a point was reached where the proportional relationship no longer held, the current increasing more rapidly than if the resistance of the film were ohmic. Furthermore, the value of the voltage corresponding to this breakdown of the linear relationship is affected by the operating conditions of the bearing.

Tudor has shown that conductance traces can be fairly well repeated, but to obtain the film thickness one must calculate it from the resistance of the oil film as obtained from the voltage current curve which must first be obtained. The method has excellent possibilities for the study of lubrication phenomena, but in its present form it has not been possible to correlate the film thickness against the Sommerfeld variable due to the necessity for more rigid control of operating conditions.

Allen (21) used the method of applying an electrical potential, between the bearing and shaft, sufficiently high to rupture the oil film. The breakdown voltage would thus be related to the minimum film thickness. For the measurements, an audio-frequency oscillator was used as the voltage source. The breakdown voltage was measured by a cathode-ray oscilloscope which was connected together with the oscillator as shown.

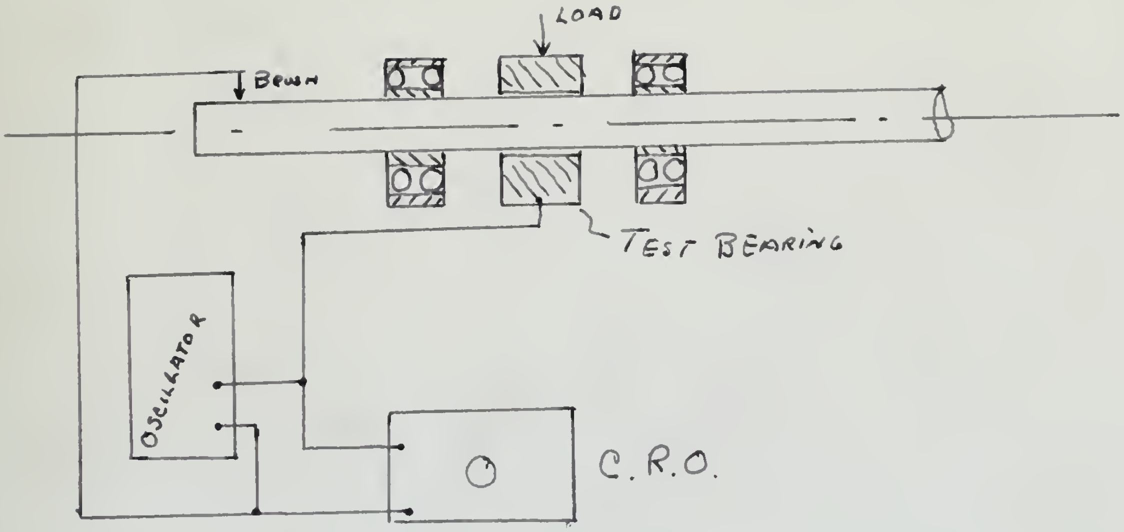


Fig. 2. Schematic of Allen's Setup

The minimum film thickness was then calculated by knowing the breakdown voltage and using an assumed value of the dielectric constant of the lubricating oil.

Shifflette (22) tried two methods of approach, one the measurement of the capacitance between the journal and bearing, the other measuring the voltage that would cause dielectric breakdown in the oil film. In both cases he used the bearing and journal as electric contacts or plates. His determination of film thickness was to calculate it from an assumed value of dielectric strength of the lubricating oil, knowing the measured capacitance in the one case and the impressed voltage that would cause

breakdown in the second. For measurement of capacitance, he used a simple Wien bridge with the capacitance between the bearing and journal being the unknown capacitance. Potential was applied to the bridge by an audio-frequency oscillator, with earphones being used to detect the minimum balance.

Vieweg (23, 24) developed two optical methods, one utilizing the pin-wheel effect due to a revolving screen on the end of the shaft, the other was based on the diffraction of a tangential ray of light.

Wolff (25) used an interference method in which a parallel beam of light of homogeneous wave length is directed into the small clearance between a blade and the oil film. To each magnitude of the clearance a definite interference corresponds, which is measured on a screen as a distance of interference fringes from the most brilliant middle fringe.

An interesting method of journal observation was used by Newkirk and Grobel (26). To accurately observe the behavior of the journal, the shaft was provided with a stiff projection. To increase the refinement of observation, the end of the projection was provided with a recess into which a 1/16 inch steel ball was set and centered with small screws. This ball acted as a convex mirror of small radius to give a virtual image of the crater of a small direct-current arc lamp. Since the diameter of the ball is small compared with the distance from the light source, the position of the virtual image relative to the ball center changes very little with small movements of the ball. A combined microscope and camera was used to observe and record the motion of the ball. The instrument was calibrated by determining the movement of the recorded light trace for a given shaft displacement.

Gregory (27) describes a method that has been used in the determination of very thin films on plane sliders in which the transfer of radioactivity from one metal through the film to the other surface was used. The deposit of radioactivity being dependent upon the thickness of the oil film and time. However, it is doubtful if a like method could be used with bearings, due to the operating characteristics.

COMPARISON OF EXPERIMENTAL RESULTS WITH THEORETICAL

The significance of any experimental result can only be fully appreciated if the fundamental conditions associated with the film lubrication of curved surfaces are clearly understood. Film lubrication must not be confused with boundary, solid film, or greasy lubrication in which the bearing surfaces are separated by an extremely thin film and no actual flow takes place. The viscosity of the lubricant and the relative movability of the surfaces are the controlling factors (8). The conditions are physical and mechanical rather than chemical, with adhesion still having an important influence.

For the purpose of comparing the various results of investigators, it is felt that the best method of approach is that of dynamic similarity (9), that is, two journal bearings are dynamically similar if they are geometrically similar and operating with equal values of some operating variable such as $\mu N/P$, where N is the number of revolutions per unit time, P the load per unit projected area, and μ the viscosity. Proceeding further with dimensional reasoning we arrive at $h_{min}/c = \psi(\mu N/P, C/D, L/D)$ where C is the diametrical clearance, D is journal diameter, L is bearing length, c is radial clearance, and h_{min} is the minimum oil film thickness. This relationship will remove the requirement of geometrical similarity as far as clearance-diameter and length-diameter ratios are concerned. For this study, it is the writer's intention to use curves of h_{min}/c (dimensionless) against the Sommerfeld variable $(D/C)^2 \mu N/P$ for corresponding values of L/D and arc subtended by the bearing. The above curves will be compared with the corresponding theoretical values as given by Boyd and Raimoni (28).

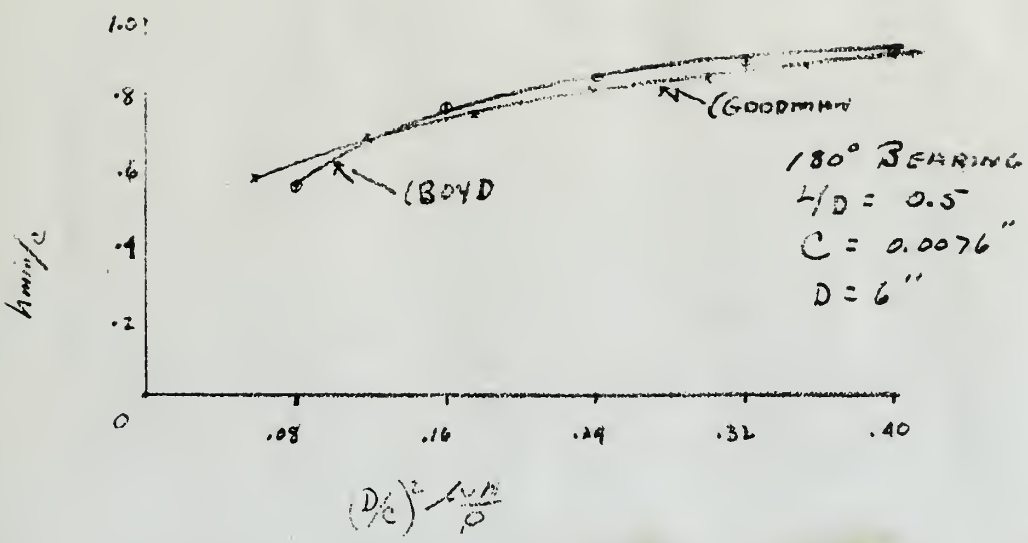


Fig. 3. Comparison of Goodman's Results With Theoretical

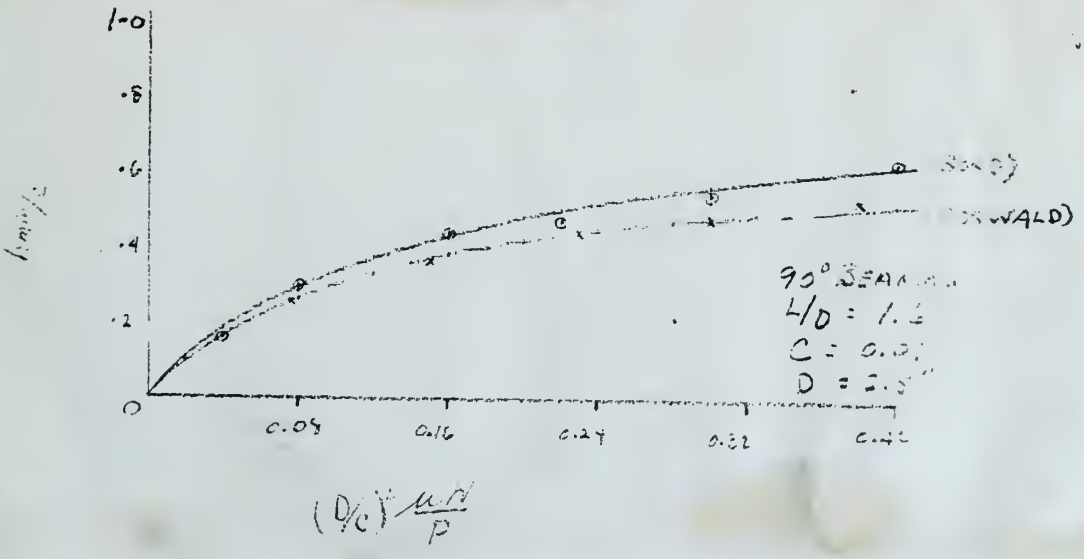


Fig. 4. Comparison of Boswald's Results With Theoretical

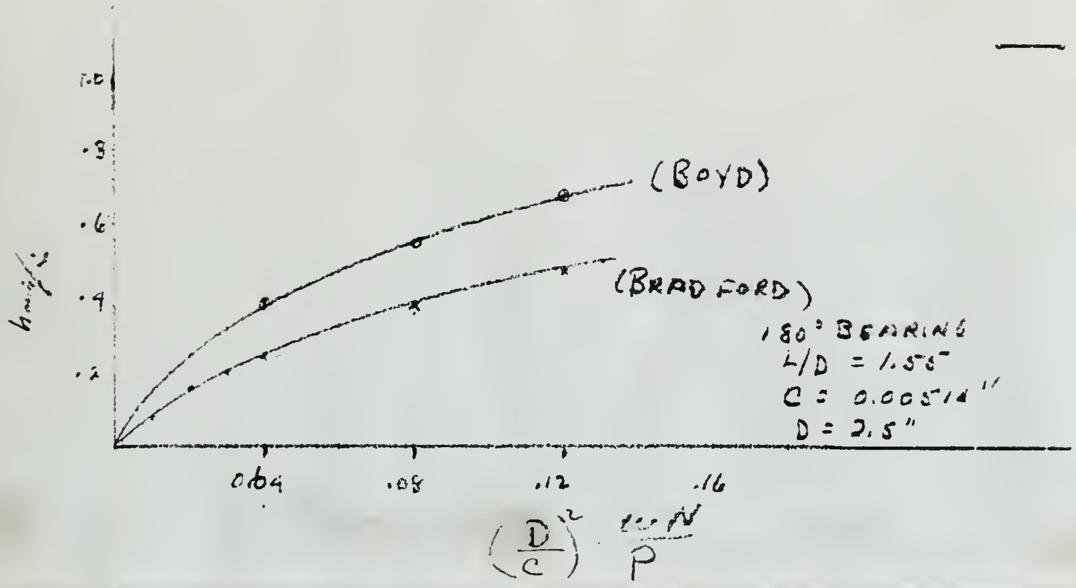


Fig. 5. Comparison of Bradford's Results With Theoretical

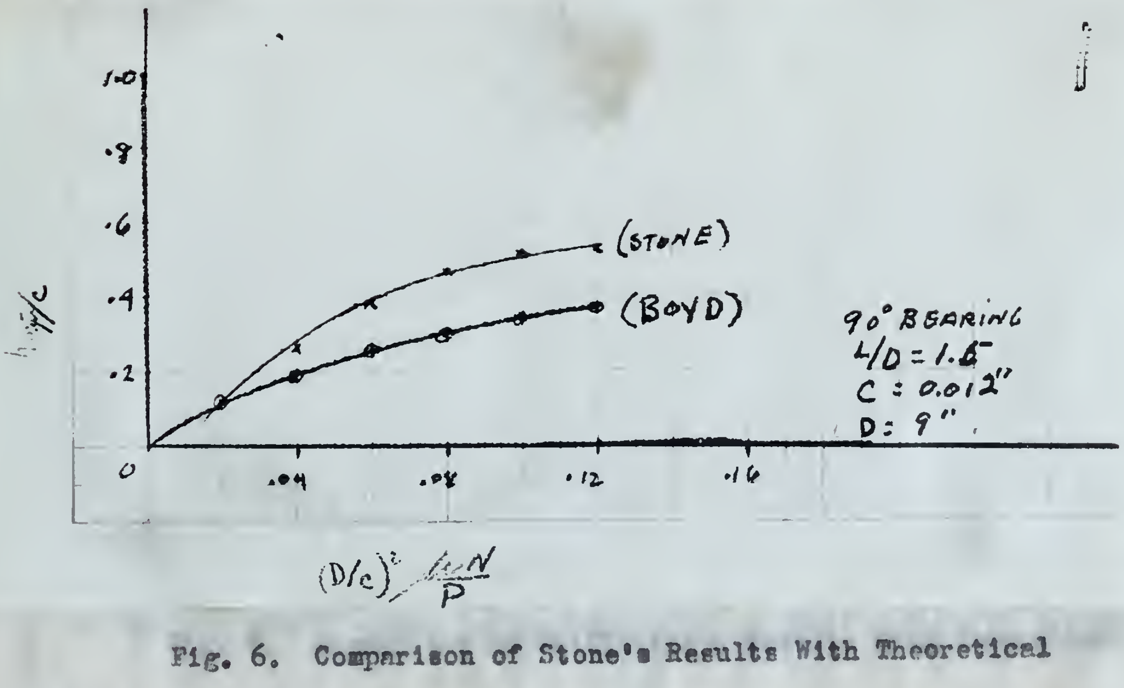


Fig. 6. Comparison of Stone's Results With Theoretical

minimum thickness of the oil wedge. Using the first approach, Stone has succeeded in making some apparently accurate measurements of the motion of the journal center. He has done this by simultaneously measuring the horizontal and vertical motion of the shaft center by measuring the voltage variations with his electromagnetic system. His measurements agree fairly well with the results obtained by the classical theoretical approach.

Greengough has used another electromagnetic system and has incorporated into it an indicator which is designed to picture the shaft center on a cathode-ray scope as an illuminated spot. He also has superimposed a scale over the scope which will read the minimum thickness and its orientation directly. It should be pointed out that this method also gives the shaft center position. Greengough's instrument has not yet proved to be quantitative.

Simons has incorporated the so called capacitive micrometer which was originally designed to check the rotation of lathe spindles. Basically it attempts to picture the shaft center on a cathode-ray scope. His results give an excellent picture of the movements of the shaft center, however, it must be said that his results are no more in agreement with the theoretical values than other methods in regards to minimum film thickness.

From the second, or more direct approach, Allen and Shifflette have used the principle that the oil film will breakdown at its thinest point when subjected to an electrical potential between the journal and bearing. If consistent results could be obtained from this method, it would possibly give the best results of all methods. However, to obtain the minimum oil film thickness, one must calculate it from the dielectric

strength of the oil in use. The exact value of the dielectric constant of very thin oil films in bearings which are subjected to high pressures, high temperatures, and enormous rates of shear will not bear any relation to test results in a standard cell since wide temperature and pressure changes have an appreciable effect upon the dielectric constant.

The conductance method as used by Tudor and the capacitance method used by Shifflette also use the bearing and journal as components of an electrical circuit. They make the assumption of constant geometry and also rely on computation of the film thickness from constants of the oil which are considered constant but which do not necessarily remain so, but change with the operating conditions of the bearing.

The writer feels that although the methods using the bearing and journal as parts of an electrical circuit are not quantitative at the present time for determining the minimum oil film thickness, they are still very useful in bearing study, particularly from the standpoint of predicting failure (21), since with these methods one is enabled to predict seizure a considerable time before any other indications of failure are observed. In this connection, it could conceivably be used as a method of obtaining the cause of the first of the train of circumstances which lead to bearing failure.

The writer feels that the method as described by Greengough, (19) when it proves to be quantitative, should probably be the preferable method of those reviewed to be used by future investigators because of its simplicity in operation and the fact that it should give the shaft center eccentricity and angular orientation directly. However, one must still remember the limitations of this method as pointed out earlier.

PROPOSED METHOD OF SHAFT ECCENTRICITY DETERMINATION

(ASSUMING THAT THE SHAFT COMES TO AN EQUILIBRIUM POSITION)

In proposing a new experimental method of determining shaft eccentricity or minimum film thickness, it is the writer's intention to recommend a method which could be used either for measurements on an actual operating bearing or on a test stand in conjunction with purely experimental bearing work. Under these conditions very high rotation speeds can be expected, therefore, it is felt that there should be no connection to the shaft itself nor should the test apparatus affect the bearing performance. Also, it is the writer's opinion that the measuring system should have sufficient damping to prevent impulses of a small vibratory nature from confusing the actual observation procedures.

The basic instrument to be used is of the new pneumatic type (29) in which the pressure between a fixed orifice (G) and a variable orifice (S) is a function of the effective size of the variable orifice.

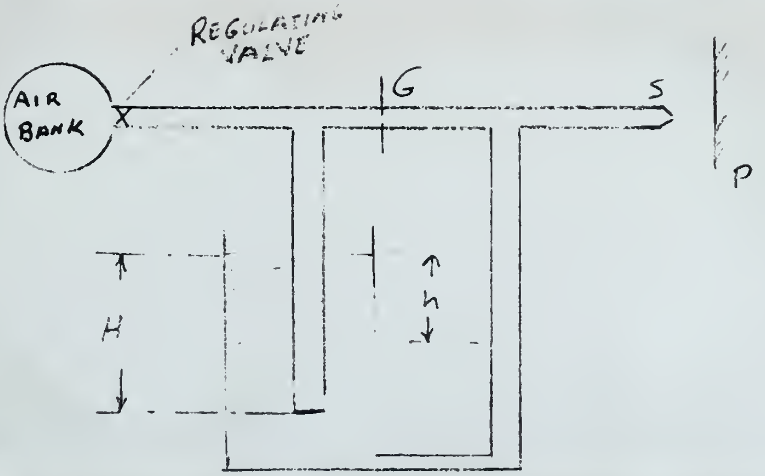
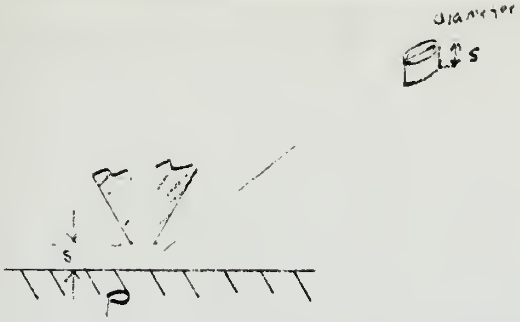


Fig. 9. Schematic Diagram of Pneumatic Apparatus

The size of the orifice G is constant while the effective area of the variable orifice is proportional to the surface area between the orifice face and the plate P:



The governing equation for this apparatus being $h = \frac{H}{4 + \frac{S^2}{G^2}}$ where h and H are manometer heights as shown in Figure 9, G is the effective orifice size of the fixed orifice, and S is the effective area of the variable orifice.

For determining the shaft eccentricity of an operating bearing, two of the above gages would be required — one for horizontal measurements, the other for vertical measurements. The gages would be secured to the bearing (B) and directed toward the journal (J) as shown in Figure 10.

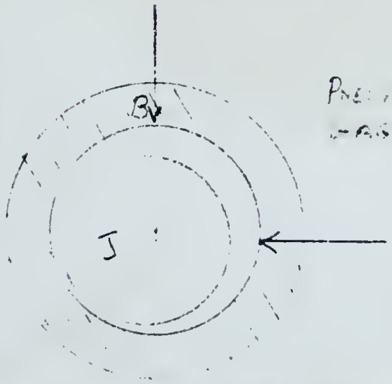
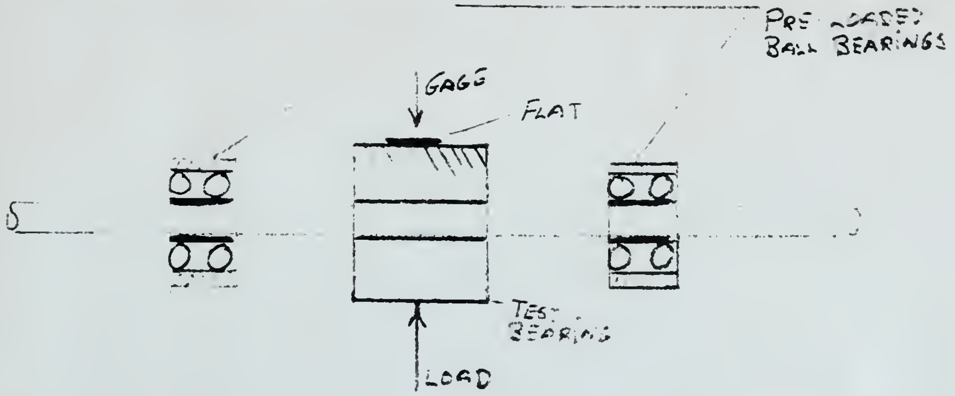


Fig. 10. Orientation of Pneumatic Gages to Shaft

Before using these gages to determine the shaft eccentricity one must first obtain a calibration curve. It is recommended that this be done by measuring the actual separation between the variable orifice face to the shaft by an optical interferometer. This calibration would not be performed on the shaft but on a like shaft which is held stiffly in place by pre-loaded ball bearings and which is being rotated during the calibration. This is necessary because viscous effects will change the calibration somewhat. Once the calibration curve is obtained, it is a straightforward matter to measure the position of the shaft with respect to the two mounted gages.

For purely experimental bearing determinations the only change to be made is to have the gages mounted in a cradle carried by the shaft and have the jets impinge on flat plates mounted on the bearing.



For this arrangement the calibration curve must be redetermined since the flat plates are stationary.

The primary advantages of this system are: first, small vibratory motions are damped out in the measuring tubes leaving one with the essential measurements that are desired, and second, the magnification factor is quite high with a single gage and can be doubled if desired with a differential type of arrangement.

SECOND PROPOSED METHOD OF FILM THICKNESS DETERMINATION

Since the calibration of any measuring system to be used for dynamic measurements is, at best, extremely difficult, it is the writer's intention to devise a scheme for purely experimental determinations, which will require no calibration once the wave length of the light used is known.

Essentially, this apparatus would consist of a quartz bearing model, a very accurately ground and polished shaft, light sources, mirrors and lenses necessary for focusing the light, and a counting mechanism to count the firing shifts at a reflected interferometer pattern.

To measure the film thickness, one would pass two beams of monochromatic light at right angles through the quartz bearing — the inside surface of which has been silvered — to the shaft. The light incident on the shaft would be reflected to the bearing surface where an interference pattern would be pictured.

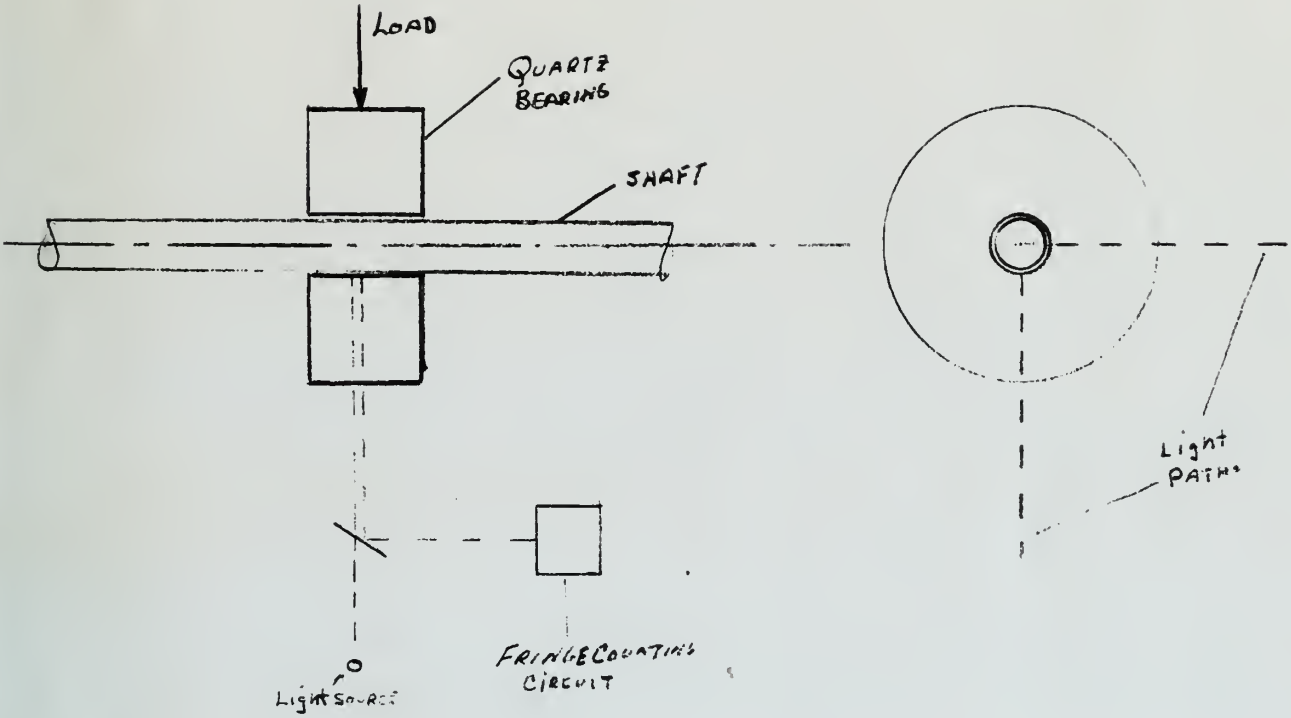


Fig. 11. Schematic Diagram of Proposed Optical Method

As the surface of the shaft moves toward or away from the bearing surface, the interference pattern would shift –– causing a fringe to go from light to dark for the movement of one-half wave length.

The fringe pattern would be viewed through two narrow slits, which are spaced a distance apart slightly less than an integral multiple of the actual distance between the fringes. The light from each slit would fall upon its own photo-cell, the output from these photo-cells would be fed into a counting circuit, as shown in Figure 12. Essentially, this circuit arrangement will give an output at the recorder of the algebraic sum of the fringe shifts; that is, if the fringes are moving in such a direction as to cause light to be incident first upon photo-cell Number 1 and then on Number 2 -- that is, a 1-2 trigger -- the output would be the sum of the triggered pulses; on the other hand, a 2-1 trigger would be subtracted leaving the algebraic sum of the number of half wave lengths motion of the shaft with respect to the bearing.

In operation, one would start from zero at a known shaft position -- relative to the bearing -- when the shaft is stopped. Then with the number of wave lengths motion (by two of the subject gages - one for vertical measurements, the other for horizontal measurements) from the known position, one is enabled to plot the position of the shaft at any time.

The STEP CHARGER and AMPLIFIER arrangement could possibly be a modification of the radio altimeter. The recorder could be one of several types, preferably a brush type, but could even be an indicating meter.

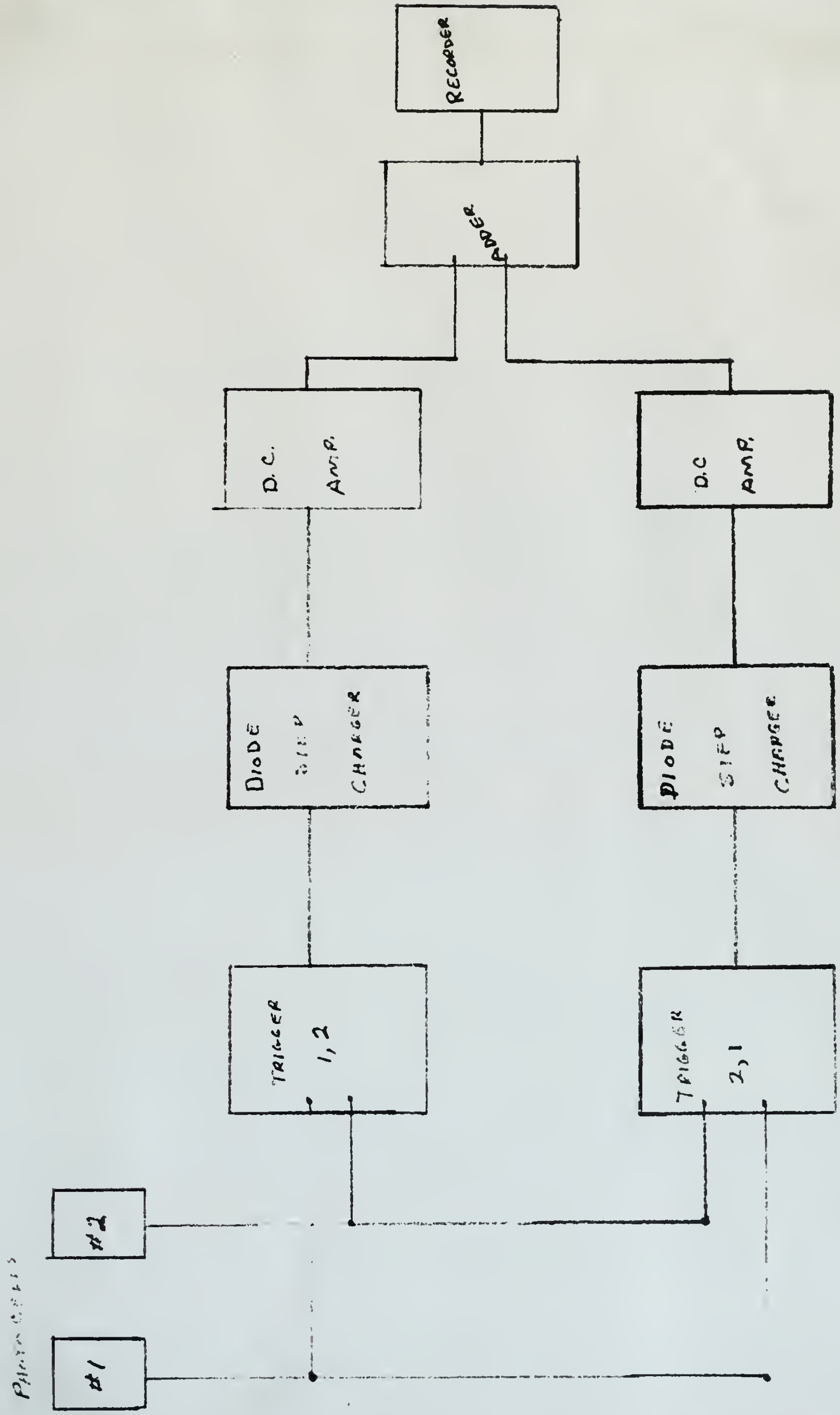


Fig. 12 Block Diagram of Fringe Counting Circuit

CONCLUSIONS

In conclusion it must be said that the direct determination of the minimum oil film thickness is very difficult if not impossible. Several methods have been devised which give the motion and position of the shaft center. These results, particularly those of Simons and Stone, have proved that the shaft center will move somewhat as predicted by the hydrodynamic lubrication theory. However, due to the fact that the geometry of the bearing is not constant, but varies considerably (in comparison with the oil film thickness) due to local elastic deformation, thermal expansion, shaft deflection through the length of the bearing, and the surface roughness of the shaft and bearing, lends to the failure of any attempt at determining the minimum oil film thickness by making measurements outside the bearing.

If some definite knowledge of the dielectric strength of the lubricating oil under the conditions it operates in a bearing could be obtained, conclusive results could be obtained by some method which used the dielectric breakdown of the film since this is the most direct attempt at measuring the film thickness.

BIBLIOGRAPHY

1. Tower, B., Proceedings of the Institution of Mechanical Engineers, 1885.

2. Reynolds, O., The Theory of Lubrication and its Application to Beaucamp Tower's Experiments, Phil. Trans. Royal Soc., Series A, 1886, part 1.

3. Sommerfeld, A., Zur Hydrodynamischen Therieder Schmiermittel reibung, Zeit. Math., Phys., Bd. 50, 1904, pp 97-155.

4. Kingsbury, A., Optimum Conditions in Journal Bearings, Trans. A.S.M.E. RP-54-8, 1932, pp 149-165.

5. Musckat, M. and Morgan, F., Studies in Lubrication, Journal of Applied Physics, vol. 9, 1938, pp 393-409; vol. 10, 1939, pp 46-61; vol. 10, 1939, pp 398-400.

6. Cameron, A. and Wood, Mrs. W. L., The Full Journal Bearing, Proc. of the Institution of Mechanical Engineers, vol. 161, 1949, pp 50-64.

7. Waters, E. O., Characteristics of Centrally Supported Journal Bearings, Trans. A.S.M.E., vol. 64, 1942, pp 711-719.

8. Boswall, R. O., and Brierley, J. G., The Film Lubrication of the Journal Bearing, Proc. Inst. Mech. Eng., vol. 122, 1932, pp 423-569.

9. Hersey, D. M., Theory of Lubrication, John Wiley and Sons, Inc., 1936, chapter IV, p 70.

10. Gümbel, L., Einfluss der Schmierung auf die Konstruction. Jahrbuch der Schiffbautechnischen Gesellschaft, 1917.

11. Stoney, G., Boswall, R. O., and Massey, J., The Thickness and Resistance of Oil Films in High-Speed Bearings. Engineering vol. 113, 1922, pp 249-250.

12. Mars, G. H. and others, Some Experiments on Oil Films in Complete Cylindrical Bearings, Stanford University Press, 1932, pp 110.

13. Goodman, J., An Experimental Determination of the Distribution and Thickness of the Oil Film in a Flooded Journal Bearing, Proc. Inst. Civil Eng., vol. 226, pt 2, 1929, pp 242-268; vol. 233 pt 1, 1933, pp 244-266, 289-322.

14. Bradford, L. J., and Davenport, C. C., Characteristic Curves for Fluid Film Lubricated Journal Bearings, Refrigerating Engineering, vol. 24, 1932, pp 343-347.

15. Kluge, J. and Linckh, H., Piezoelektrische Messung Mechanischer Grösse, Forschung No. 2, 1935.

16. Stone, M., Film Lubrication in Sleeve Bearings, Jour. of Applied Mechanics, vol. 2, 1935, pp A:59-64, vol. 3 pp A:31-34.

17. Stone, J. M. and Underwood, A. F., Load Carrying Capacity of Journal Bearings, Trans. S. A. E., vol. 1, 1947, pp 56-57.

18. Simons, E. E., The Hydronamic Lubrication of Cyclically Loaded Bearings, Trans. A. S. M. E., vol. 72, 1950, pp 805-816.

19. Greengough, M. L., Oil Film Thickness Indicator for Journal Bearings, Trans. American Institute of Electrical Engineers, vol. 67, pt. 1, 1948, pp 589-595.

20. Tudor, G. K., An Electrical Method of Investigation in a Journal Bearing, Journal of Scientific and Industrial Research, Australia, vol. 21, 1948, pp 202-209.

21. Allen, C. M., The Dielectric Strength of Oil Film in Plain Bearings, Mechanical Wear, Burwell, American Society of Metals, 1950, p 181.

22. Shifflette, W. M., Determination of Film Thickness by Electrical Means, Unpublished thesis, U. S. Naval Postgraduate School, 1949.

23. Vieweg, V., Optische Messägerate Zur Bestimmung der Dicke der Oelschicht in Lagern, Petroleum Zeitschrift, 1922.

24. Vieweg, V., Bericht Über die oel und Lagerversuche im Maschinen Laboratorium der Physikalisch Technischer Reichsanstalt, Maschinenbau, Bd. 12, 1923, pp B:131-133.

25. Wolff, R., On the Oil Film in Sleeve Bearings and its Measurement by Means of Interference, Forschungsarbeit, Heft No. 308, 1928.

26. Newkirk, B. L. and Grobel, L. P., Oil Film Whirl, a Non-Whirling Bearing, Trans. A.S.M.E., vol. 56, 1934, pp 607-615.

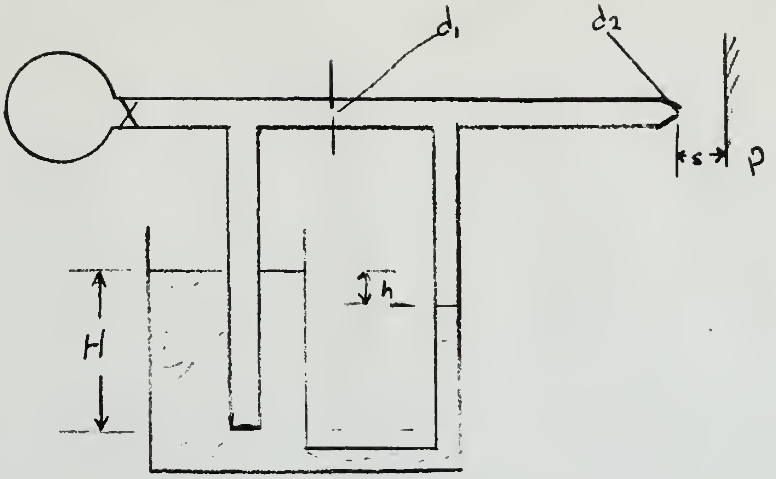
27. Gregory, J. N., Radioactive Tracers in the Study of Friction and Lubrication, Nature, vol. 157, April 1946, p 443.

28. Boyd, J. and Raimondi, A. A., Applying Bearing Theory to the Analysis and Design of Journal Bearings, Journal of Applied Mechanics, vol. 18, ~~1941~~, pp 298-316.
1951

29. Windlack, W. A., A Versatile Pneumatic Instrument Based on Critical Flow, Review of Scientific Instruments, vol. 21, 1950, pp 25-30.

APPENDIX A

Basic calculations for pneumatic type gage.



Starting with the continuity equation $C_1 A_1 \sqrt{H-h} = C_2 A_2 \sqrt{h}$

$A_1 = \frac{\pi d_1^2}{4}$; $A_2 = \pi d_2 s$; ASSUME $C_1 \equiv C_2$

then $d_1^4 (H-h) = 16 d_2^2 s^2 h$

$h = H\left[\frac{d_1^4}{d_1^4 + 16 d_2^2 s^2}\right]$, letting $Q = \frac{16 d_2^2}{d_1^2}$ - - - - (1)

$h = \frac{H}{1+Qs^2} = H\left[1+Qs^2\right]^{-1}$ - - - - - - - (2)

differentiating with respect to s

$\frac{dh}{ds} = \frac{-2QHs}{(1+Qs^2)^2}$ - - - - - - - - - - - (3)

using the terminology that $\frac{dh}{ds} = m_s$

$m_s = \frac{-2QHs}{(1+Qs^2)^2}$

differentiating once more to find the rate of change of m_s

$\frac{dm_s}{ds} = \frac{d^2h}{ds^2} = 2QH\left[\frac{3Qs^2-1}{(1+Qs^2)^3}\right]$ - - - - - (4)

For maximum magnification $\frac{dM_s}{ds} = 0$ therefore $3QS_m^2 - 1 = 0$ or $QS_m^2 = 1/3$ --- (5)

or for maximum magnification $(M_s)_{max}$; $Q = \frac{1}{3S_m^2}$

where S_m is the particular value of s for maximum magnification

$$\therefore \frac{16 d_2^2}{d_1^4} = \frac{1}{3S_m^2} \; ; \quad S_m^2 = \frac{d_1^4}{48 d_2^2} \; ; \quad S_m = \frac{1}{\sqrt{48}}\left[\frac{d_1^2}{d_2}\right]$$

this final relation gives the relationship between the ratio of the orifice diameters, but one must first find S_m . To do this we will proceed with the critera that we desire the maximum minimum magnification over the entire range of measurement. From equation (2) $h = \frac{H}{1+Qs^2}$

but from equation (3) $QS_m^2 = 1/3$ therefore $h_m = \frac{H}{1+1/3} = \frac{3H}{4}$

where h_m is the manometer height when the variable orifice is at a distance S_m from the plate. Therefore $(M_s)_{max} = -\frac{3}{8}\frac{H}{S_m}$ ---- (6)

From equation (3) and using $Q = \frac{1}{3S_m^2}$ $\quad M_s = -2\left(\frac{1}{3S_m^2}\right)Hs \Big/ \left(1+\frac{s^2}{3S_m^2}\right)^2$

rewriting as $\frac{M_s}{H/S_m} = -\left(\frac{2}{3}\right)\left(\frac{s}{S_m}\right) \Big/ \left(1+\frac{s^2}{3S_m^2}\right)^2$ and letting the dimensionless variable $\frac{s}{S_m} = x$; $M_s \Big/ \frac{H}{S_m} = \frac{-6x}{(3+x^2)^2}$

dividing both sides of the above by $(M_s)_{max} \Big/ \frac{H}{S_m}$ we arrive at

$$\frac{\frac{M_s}{H/S_m}}{\frac{(M_s)_{max}}{H/S_m}} = \frac{16x}{(3+x^2)^2} \quad \text{or} \quad \frac{M_s}{(M_s)_{max}} = \frac{16x}{(3+x^2)^2} \quad ---- \quad (7)$$

We can now make a dimensionless plot of $M_s/(M_s)_{max}$ against x.

We are now ready to introduce the range over which we wish to use the

instrument $\Delta s =$ some known real number.

Proceeding with a numerical analysis to determine s_m. To do this we pick a series of values of $\Delta s/s_m$ -- from the curve we can obtain the corresponding value of $M_s/(M_s)_{max}$ this will give us $M_s = (M_s)_{max}\,(\text{some Number})$ but from equation (6) $(M_s)_{max} = -\frac{3}{8}\left(\frac{H}{s_m}\right)$. These two relations will give us M_s in terms of H and s.

But since we started this analysis with a picked value of Δs or the range of the measurement desired and we have picked various values of $\Delta s/s_m$, that is $s_m = \frac{(\text{Number})}{\Delta s}$ this enables us to find a numerical value of M_s.

Example: $\Delta s/s_m = 1$; from curve $\frac{M_s}{(M_s)_{max}} = 0.836$

$$M_s = 0.836\,(M_s)_{max} = \left(-\frac{3}{8}\right)\left(H/s_m\right)(0.836); \quad \text{but } s_m = \Delta s$$

$$\therefore M_s = -\frac{3}{8}\,\frac{H}{\Delta s}\,(0.836)$$

Since both H and Δs are fixed numbers M_s has some absolute magnitude. Going through this same procedure for many values of $\Delta s/s_m$ we can pick the value of $\Delta s/s_m$ which will give a maximum minimum value of M_s over the range desired. Doing this we find --

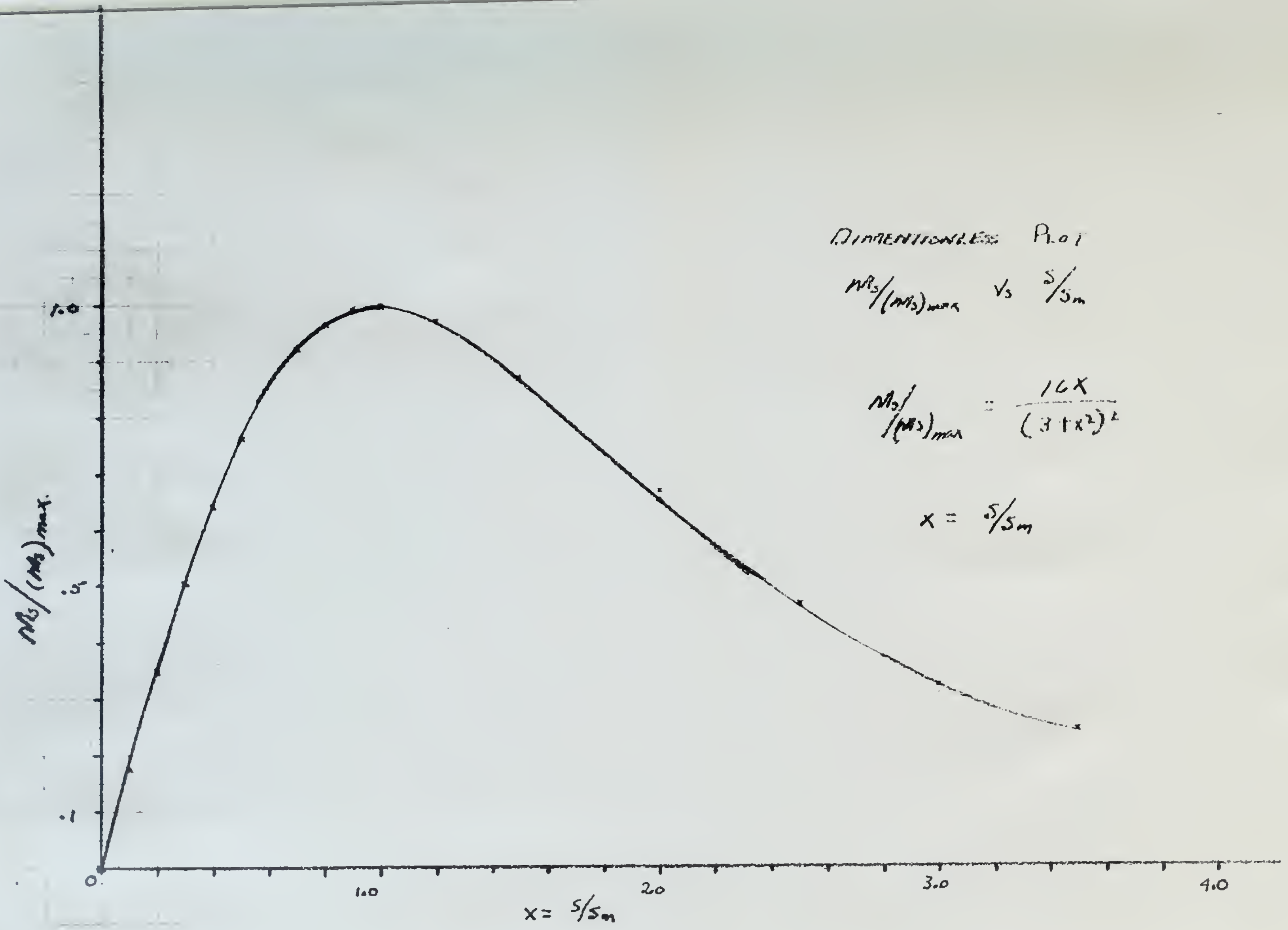
$s_m = 0.52\,\Delta s$

$s_1 = 0.33\,s_m$ --- $h = 0.966\,H$

$s_2 = 2.26\,s_m$ --- $h = 0.37\,H$

$M_s = -\left(\frac{3}{8}\right)(.553)\left(H/s_m\right)$

$(M_s)_{max} = \left(-\frac{3}{8}\right)\left(H/s_m\right)$


DIMENTIONLESS PLOT
Ms/(Ms)max Vs S/Sm
Ms/(Ms)max = 16X/(3+x²)²
x = S/Sm
Ms/(Ms)max
1.0
.5
.1
0
1.0
20
3.0
4.0
x = S/Sm